Differentiation:
Partial Differentiation

Suppose $f(x, y)$ is a function of two variables x and y.
The partial derivative of f with respect to x is written as $\frac{\partial f}{\partial x}$ or f_x.
This is found by differentiating f with respect to x, with y held constant.

Similarly, the partial derivative of f with respect to y is written as $\frac{\partial f}{\partial y}$ or f_y.
This is found by differentiating f with respect to y, with x held constant.

Examples

$$f(x,y) = x^2 + y^3 : \quad \begin{cases} f_x = 2x + 0 = 2x, \text{ since } y \text{ is constant.} \\ f_y = 0 + 3y^2 = 3y^2, \text{ since } x \text{ is constant.} \end{cases}$$

$$f(x,y) = x^2 y^3 : \quad \begin{cases} f_x = 2xy^3, \text{ since } y \text{ is constant.} \\ f_y = 3x^2y^2, \text{ since } x \text{ is constant.} \end{cases}$$

$$f(x,y) = \log_e\left(1 + x^2 y\right) : \quad \begin{cases} f_x = 2xy \cdot \frac{1}{1+x^2y} = \frac{2xy}{1+x^2y}, \text{ since } y \text{ is constant.} \\ f_y = x^2 \cdot \frac{1}{1+x^2y} = \frac{x^2}{1+x^2y}, \text{ since } x \text{ is constant.} \end{cases}$$

$$f(x,y) = e^{2x+y^2} : \quad \begin{cases} f_x = 2e^{2x+y^2}, \text{ since } y \text{ is constant.} \\ f_y = 2ye^{2x+y^2}, \text{ since } x \text{ is constant.} \end{cases}$$

$$f(x,y) = x^4y^5 - x^2 + y^2 : \quad \begin{cases} f_x = 4x^3y^5 - 2x, \text{ since } y \text{ is constant.} \\ f_y = 5x^4y^4 + 2y, \text{ since } x \text{ is constant.} \end{cases}$$